DAMPING OF A SHOCK WAVE ON THE COLLISION OF PLATES

A. P. Rybakov

UDC 534.22

INTRODUCTION

The damping of a plane shock wave was treated in [1, 2] in the one-dimensional case and in the hydrodynamic approximation. The formula for damping of a shock wave was obtained in the form of a finite-difference equation in [1], based on the hypothesis that energy decays in the zone of the shock wave. Although this hypothesis does not contradict the conditions of dynamic consistency at the shock front it has no strict justification. However, the damping formula is in satisfactory agreement with known experimental data. Formulas for the shock front and the form of the momentum as functions of time are obtained in [2] on the basis of Friedrichs method for the case of a striker and a barrier of the same material. A simpler method is given below for determining the position of the shock front as a function of time.

We shall use the same assumptions as in [2] to treat the propagation of a shock wave created in an obstacle by the blow from a plate. We shall make the approximation of assuming that the compression shock is an isentropic process. We shall treat the wave propagation in the hydrodynamic approximation without taking rigidity, viscosity, and thermal conductivity into account. Moreover, for simplicity we shall assume initially that the striker and object struck are made of the same material. The equation of state of this material can be represented by the equation [3]

$$p = \frac{\rho_0 c_0^2}{n}(\sigma^n - 1), \tag{1}$$

where p is the pressure, ρ is the density, c is the velocity of sound, n is a constant, and $\sigma = \rho/\rho_0$ is the compression. The subscript 0 indicates that the quantity refers to the initial state. We shall treat the process in the coordinates of space x and time t adopted in gasdynamics. Let the moment of collision coincide with the coordinate origin.

When the plate collides with the obstacle, shock waves propagate in both directions from the contact boundary (Fig. 1). A centered rarefaction wave propagates to the right from the point (x_n, t_n) where the shock wave exits to the rear free surface of the striker plate. Let 0 and 1 denote the state of the material in front and to the rear of the shock wave, respectively. Let 2 denote the state of the material after the rarefaction wave has propagated through it. We introduce the following additional symbols: D, the velocity of the shock wave; u, the mass velocity. The leading characteristic of the rarefaction wave overtakes the shock front at the point (x_m, t_m). For the wave propagating to the right the Riemann invariant is constant $I_- \equiv$ const [4]. In this case

$$c = [(n-1)/2]u + c_0.$$

The equation of the c_+ characteristic is

$$(x - x_n)/(t - t_n) = c_0 + [(n+1)/2]u. \tag{2}$$

The equation for the trajectory of the shock front up to the point (x_m, t_m) is the straight line $(x/t) = D =$ const, and after this point it is

$$dx/dt = c_0 + \beta u. \tag{3}$$

Here use has been made of the well-known relation between the wave velocity D and the mass velocity u, while

Chelyabinsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 147-149, September-October, 1976. Original article submitted January 27, 1976.

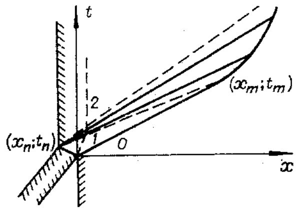

Fig. 1

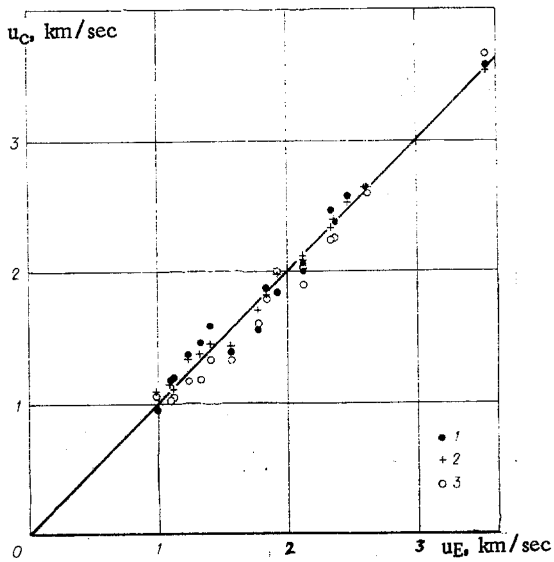

Fig. 2

c_0 and β are constant coefficients. The quantity c_0 has the physical meaning of being the initial velocity of sound in the absence of phase transition.

It can be shown that in the present approximation of an isentropic shock wave Eq. (3) follows from Eq. (1) Then

$$\beta = (n + 1)/4.$$

The fact that the linear relationship (D−u) exists in practice for condensed bodies confirms the validity of the assumption that shock-wave compression in condensed bodies is isentropic. A departure from linearity in the relation (D−u) for strong waves is a result of the fact that strong shock waves are not isentropic. From Eq. (2) we have

$$u = [2/(n + 1)] [(x - x_n)/(t - t_n) - c_0]. \tag{4}$$

Substitution of Eq. (4) in Eq. (3) gives the relation

$$dx/dt = A + B(x - x_n)/(t - t_n),$$

where

$$B = 2\beta/(n + 1) = 1/2, \quad A = (1 - B)c_0 = c_0/2.$$

The solution of this differential equation has the form

$$(x - x_n)/(t - t_n) = c_0\{1 - [1 - (x_m - x_n)/c_0(t_m - t_n)] [(t_m - t_n)/(t - t_n)]^{1/2}\}. \tag{5}$$

Equation (5) describes the trajectory of the shock front in the attenuation zone of the rarefaction wave after the point (x_m, t_m). In the range $0 \le x \le x_m$ the equation for the trajectory of the shock front is the straight line $x = Dt$, and the mass velocity behind the wave front remains constant: $u = u_1$. The magnitude of the mass velocity at the front in the attenuation zone of the rarefaction wave can be obtained from Eqs. (4), (5) as a function of time:

$$u = u_1 [(t_m - t_n)/(t - t_n)]^{1/2}. \tag{6}$$

Thus, the mass velocity at the shock front varies in inverse proportionality to the square root of time, i.e., it obeys Landau's law [5] for weak waves.

If the striker and the obstacle are made from different materials, the centered rarefaction wave from the rear surface of the striker is refracted and passes into the obstacle. In this case a new pole (x_n', t_n') can be found for the refracted rarefaction wave. If the coordinates of this pole are substituted into Eqs. (5), (6), these equations are then valid for the case where different materials collide.

The experimental results of [6] allow us to verify the validity of different damping formulas. Calculated results and experimental data [6] are compared in Fig. 2. Experimental values of the mass velocity u_E are given on the abscissa, and the calculated values u_C are given on the ordinate.

The solid line corresponds to experiment. Calculated values obtained in [1], on the hypothesis that the energy decays in the shock-wave zone, are denoted by the number 1, results determined from Eq. (6) are denoted by 2, while 3 denotes values of the mass velocity determined from a formula which, in our symbols, gives the mass velocity as an explicit function of time obtained in [2]:

$$u = c_0\{2(n+1)/[1 + E\langle(t - t_n)/(t_m - t_n)\rangle^{1/2}] + 1\} - c_1, \tag{7}$$

where

$$E = 2(n+1)/[(u_1 + c_1)/c_0 - 1] - 1.$$

The following values of n, determined from experimental pressure and compression, were taken when making calculations from Eq. (7): 4.5 for Al, 5.2 for Pb, 5.05 for Cu, and 5.86 for Fe. Values of the quantities t, x, u_1, c_1 were taken from [6], while values of c_0 for four metals were taken from [7].

It is clear from Fig. 2 that all three damping formulas give values of the mass velocity close to the experimental values. Kozlov [1] noted that the discrepancy between experimental values and those calculated from his formula was less than 13.5%. The discrepancy between experimental results and those calculated from Eq. (7) reaches 16.2%. Calculations from Eq. (6) give a discrepancy with experiment of less than 10%, and, in the majority of cases, less than 3-4%.

Thus, Eq. (6) gives a somewhat better agreement with experiment.

LITERATURE CITED

1. V. P. Kozlov, "Two cases of the propagation of a shock wave in a metal," Zh. Tekh. Fiz., 36, No. 7, 1305 (1966).
2. G. R. Fowles, "Attenuation of the shock wave produced in a solid by a flying plate," J. Appl. Phys., 31, No. 4, 655 (1960).
3. F. A. Baum, L. P. Orlenko, K. P. Stanyukovich, V. P. Chelyshev, and B. I. Shekhter, The Physics of Explosion [in Russian], Nauka, Moscow (1975), p. 268.
4. Ya. B. Zel'dovich and Yu. P. Raizer, The Physics of Shock Waves and High Temperature Hydrodynamic Phenomena, 2nd ed., Academic Press (1966-1967).
5. L. D. Landau and E. M. Lifshits, The Mechanics of Continuous Media [in Russian], Gostekhizdat, Moscow (1954), p. 460.
6. L. V. Al'tshuler, S. B. Kormer, M. I. Brazhnik, L. A. Vladimirov, M. P. Speranskaya, and A. I. Funtikov, "Isentropic compressibility of aluminum, copper, lead, and iron at high pressures," Zh. Éksp. Teor. Fiz., 38, No. 4, 1061 (1960).
7. V. N. Zharkov and V. A. Kalinin, Equations of State of Solid Bodies at High Pressures and Temperatures [in Russian], Nauka, Moscow (1968), pp. 186-189.